Potential problems with HYDRA

Basis and application of the Potential Theory for temperature conductors and ground water currents

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Potential problems

- Classical potential problem
  - Heat conduction (convection) incl. radiation border conditions
  - Ground water currents
  - Electrical and magnetical fields
  - Membrane-Solutions
  - Torsions/Shear problems at the cross-section
- However not solvable
  - Navier-Stokes Equations (Fluid-Mechanic)
  - Bi-Potential equations (Slabs, disks)
  - Transport problems (convection, aggressive substance diffusion)

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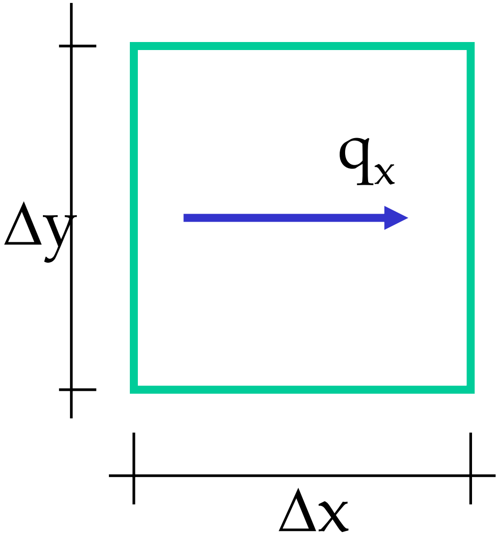
Heat conduction 2D

Searched:
Temperature field T(x,y)

Heat conductibility k

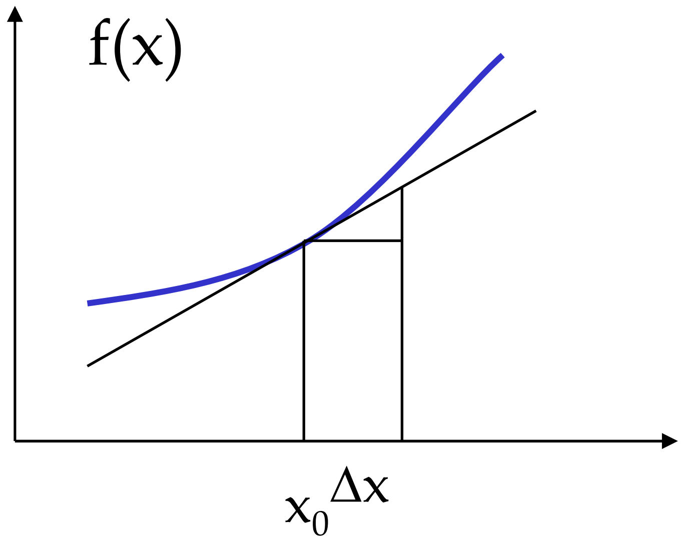
$$q_x = k \cdot \frac{\Delta T}{\Delta x} \Rightarrow q_x = k \cdot \frac{\partial T}{\partial x}$$

$$q_y = k \cdot \frac{\Delta T}{\Delta y} \Rightarrow q_y = k \cdot \frac{\partial T}{\partial y}$$



Partial Differential

Series expansion



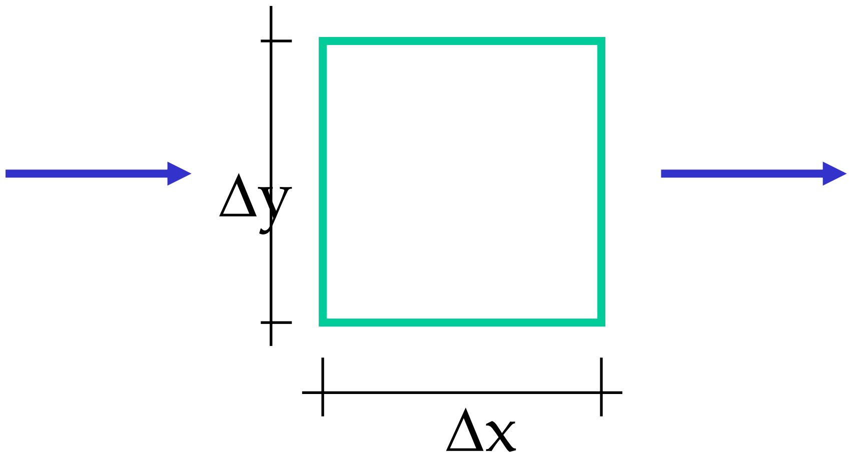
$$f(x+\Delta x) = f(x_0) + \Delta x \cdot \frac{df(x_0)}{dx} + \frac{\Delta x^2}{2!} \cdot \frac{d^2 f(x_0)}{dx^2} + \ldots$$

Taylorreihe

(*für* $\Delta x \to 0$ Any term of a higher order disappears!)

Amounts of heat

q_x



$q_x + \Delta x \cdot \frac{\partial q_x}{\partial x}$

$$\Delta y \cdot \left(q_x + \Delta x \cdot \frac{\partial q_x}{\partial x} - q_x \right) + \Delta x \cdot \left(q_y + \Delta y \cdot \frac{\partial q_y}{\partial y} - q_y \right) = 0$$

Laplace Differential equations

$$\Delta y \cdot \left(q_x + \Delta x \cdot \frac{\partial q_x}{\partial x} - q_x \right) + \Delta x \cdot \left(q_y + \Delta y \cdot \frac{\partial q_y}{\partial y} - q_y \right) = 0$$

$$q_x = k \cdot \frac{\partial T}{\partial x} \quad q_y = k \cdot \frac{\partial T}{\partial y}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(k \cdot \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \cdot \frac{\partial T}{\partial y} \right) = 0$$

$$\Rightarrow k \cdot \Delta T = 0$$

Solving the Laplace-Equation

- Analytical
  Any function of a complex variable in the Real- and imaginary part represent a solution of the Laplace DGL. (conform representation)

$$F(z) = g(z) + i * h(z)$$

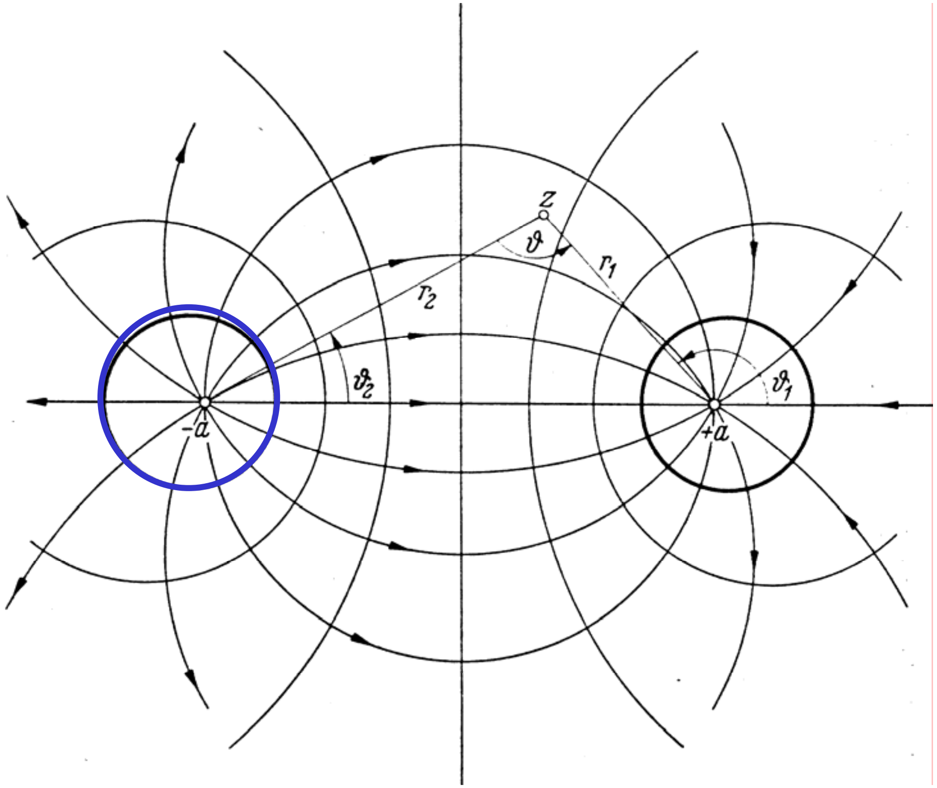
i.e.

$$Ln(z) = ln(x^2+y^2) + i * atan (y/z)$$

Two point current

- By combining the functions further solutions appear:

$$\frac{1}{2\pi}\ln\left(\frac{z+a}{z-a}\right)$$



Solving the Laplace-Equation

- Problem: Border conditions
  Apart from a few cases, it’s not possible to find simple functions which comply with the border conditions.
- Principal solution possibilities:
  Several functions are taken in order to try to comply approximately with the border conditions – either average or point-wise. (here not in deep)
  Example: Integral equation procedure

Alternative wording

- Variation computation
  Integral over the area Ω and the border Γ

$$\int\int k \cdot \left(\frac{\partial T}{\partial x}\right)^2 + k \cdot \left(\frac{\partial T}{\partial y}\right)^2 d\Omega - \int q \cdot T d\Gamma = Minimum$$

A little support:
The kinetic energy is as you know $\frac{1}{2}mv^2$
It looks as if this is also an energy term.

Finite Elements

- Solution of the variation problem
  One unknown per node: The Potential
- Analogies to the structure mechanic

– Potential	Displacement
– Gradient	Elongations
– Currents q	Stresses σ
– Amounts Q	Node loads
– Mixed border cond.	Spring element

- At least one potential value (=displacement of support) has to be known.

Mathematical terms

- **Scalable Field**
  A scalable size U, which has certain (unique) values at any point of the room, is called a scalable filed (i.e. temperature, potential, stress)
- **Vector field**
  A size u, which has been defined as vector at any point of the room (i.e. displacement, current velocitiess) is called a vector filed

Mathematical terms

- By the derivation of a scalable field one receives i.a. a vector field

$$u = gradU = \begin{bmatrix} \dfrac{\partial U}{\partial x} \\ \dfrac{\partial U}{\partial y} \\ \dfrac{\partial U}{\partial z} \end{bmatrix}$$

Mathematical terms

- A conservative or potential field is a vector filed in which the curve integral ∫udr is independent of the integration way between two points A and B.
- A conservative field is free of eddies which means the circling integral with A=B equals zero.
- However, if two sources are in the system, the circling integral equals to the sum of the sources (eddies).
- Note:
  Usually the potential is only unique when a reference value was given

Mathematical terms

- By the derivation of a vector field we receive a divergence and a rotation

$$div\ u = \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right]$$

$$rot\ u = \begin{bmatrix} \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \\ \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \\ \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \end{bmatrix}$$

Mathematical terms

- The Laplace-equation can also be written down:

$$\Delta U = \text{div grad } U$$

- **Maximum principle**
  A conservative filed always takes on the largest and the smallest potential value on the border.
- **Natural border conditions**
  On the borders where references were not given (free border) we have the border conditions that the current is vertical 0 to the border.

Heat conduction

$$div(-K \cdot \deg ree T) = q - \rho \cdot c_p \frac{\partial T}{\partial t}$$

- Potential = Temperature T
  (Kelvin or Celsius or Fahrenheit don't matter)
- Gradient = Temperature down gradient [Kelvin/m]
- Conductivity K in [W/Kelvin/m]
- Heat stream v = Conductivity * Gradient [W/m2]
- Density [kg/m^3] and specific Heat capacity c_p [W/K/kg] are combined to S [W/K/ m^3]
- Possible conversion with 1 kcal = 4186.8 Wsec

Border conditions

- Temperature $T = T_0$
- Stream line $v = - K \text{ grad } T = v_0$
  especially the adiabate border with $v^t n=0$
- Interim conditions (Interface element)
  $v = \alpha \, (T - T_u)$
- Radiation border conditions (Boltzmann-Law)
  $v = \varepsilon \, K \, (T^4 - T_u^4)$
- Temperature depending material constants

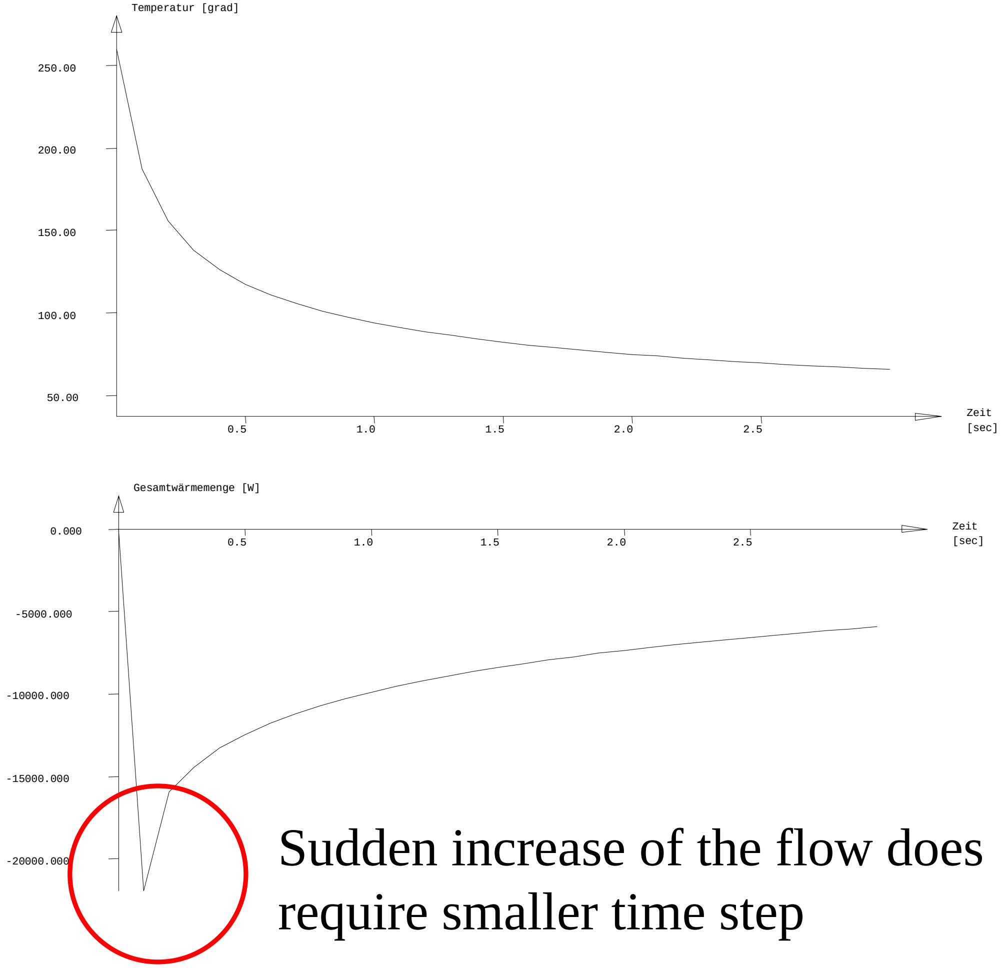
Example cooling off

- A hot block is thrown into a cooling off bath
  (HYDRA – Manual 6.2.2.)

```
KOPF ABKUEHLUNG VON 260 GRAD AUF 37 GRAD 3 SEKUNDEN
ECHO POT,STAT ; SYST DIMQ W     $ TEMPERATURPROBLEM
LF 4 ; PLF HP 260 ; STEU SCON 2
STEP 30 0.1 HOPT 10
MAT  NR   KXX   KYY    S   $ ANISOTROPE LEITFAEHIGK.
      1    34.6  6.24  242000 $ SPEZ. SPEICHERKOEFF.
RAND  TYP   VAL   VON  DELT VP  NB
      SPEZ  1360  100  GLNS 37.  1 $ UEBERG.WIDERST.
HIST H,Q  61 ; HIST QB  1 ; HIST QUAD 51
ENDE
```

- A lap (hop) is in the heat stream, therefore the time steps should be changed dynamically (HOPT).

Temperature course

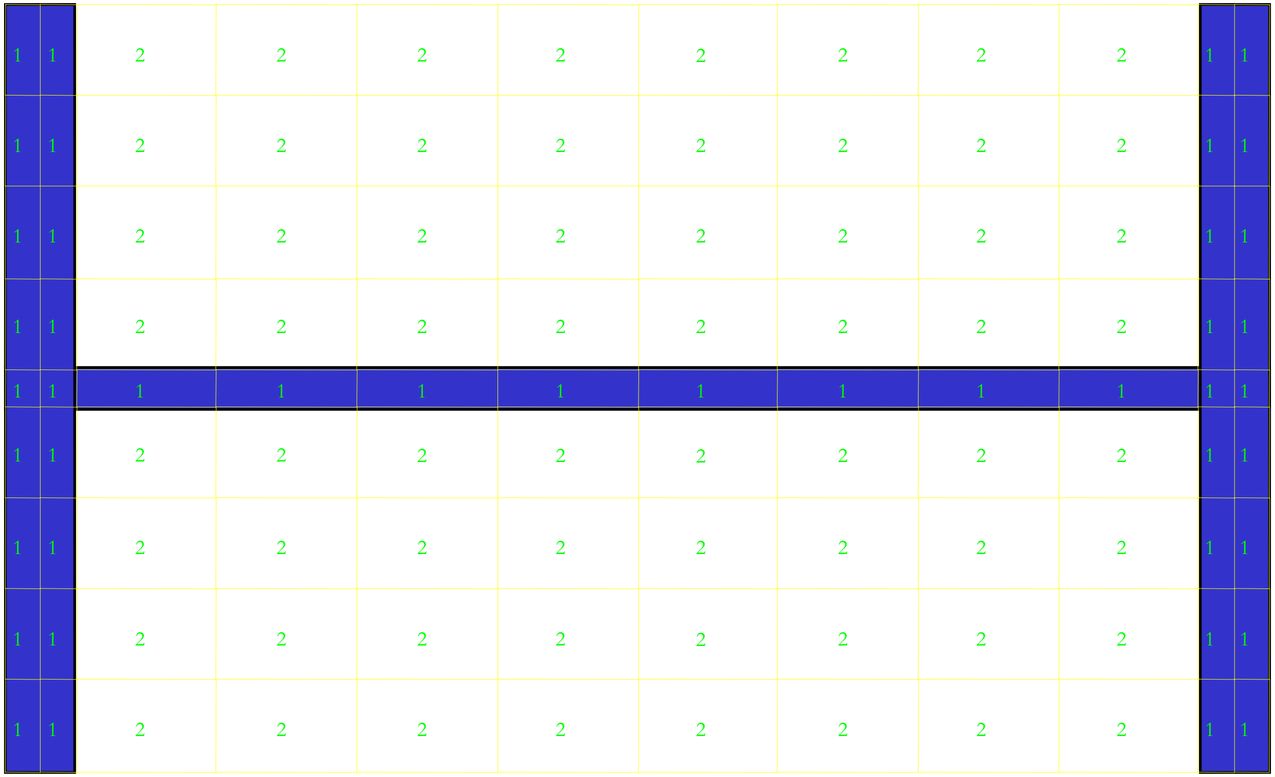


Sudden increase of the flow does require smaller time step

Example: Behavior in case of fire

- HYDRA Example 6.2.3.
- Steel looses its resistance in the case of fire.
- Therefore steel is often cased with concrete, which keeps off the heat.
- To proof the resistance the temperature course after 90 min is required (fire resistance F90)

FE-System



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Parameters for analysis

- Time (in-stationary) analysis
- Standard fire curve T=T(t)
- Radiation border condition
- Temperature depending conductivities
- Evaporation of the interstitial fluids in the concrete at 100 degrees

HYDRA - Highlights

- In-stationary analysis with automatical selection of the time step
- Continuation of primary states
  (Changing of the border conditions or re-treating of the concrete)
- Equilibrating the amounts group-wise
- Temperature depending material constants
- Radiation border conditions
- Analysis of the time course of any sizes for DYNR (GRAF-Licence)
- Taking over the loads in ASE/TALPA

HYDRA – Drawbacks

- Insolvable: Convection problems
  - Exhaust of smoke gases
  - Hot water heater
  - Up current
- Not yet solvable: Limit load of cross section
  - Special program for Geilinger round supports
  - Workaround:
    different materials in AQUA
  - FEM-Cross section in AQUA/AQB planned

Ground water currents

$$div(-K \cdot \deg ree\, H) = q - S \cdot \frac{\partial H}{\partial t}$$

- Potential = standpipe level H [m] = Sum of
  - Geodatical height z
  - Pressure height p/γ
  - velocity height $v^2/2g$

  Name due to measurability of free water levels
- Gradient = Potential down gradient [m/m]
- Conductivity K acc. to Darcy in [m/sec]
- Velocity v = Conductivity * Gradient [m/sec]
- Accumulation coefficient [1/m]

Accumulation coefficient S

- Indicates how much fluid can be stored in a volume if the pressure changes
  - Portion due to compression of the aggregate structure
  - Portion due to compression of the fluid
  - Portion due to filling up the dry cavities
- Last portion, If present, about 3 to 5 to the tenth power larger
- Extreme complex task: This portion should be analyzed in 3D.

Potential

- Current takes place from the high to the low potential
- Potential is constant for any meshed borders

Dead load direction

- Factor has effect on the geodatical height this means: that is the potential has the same value!

Border conditions

- Potential value $H = H_0$
- Stream line $v = v_0$
  Especially the impermeable border with $v^t n=0$
- Percolation area $H = z$ or $p = 0$
  (automatical control $Q<0$ in nodes)
- Free surface (Position is not known in advance)
- Leakage border conditions (Interface element)
  $$v = \alpha (H - H_u)$$
- Integral well conditions $\Sigma Q = Q_0$
- Pressure depending material constants to describe the partially saturated currents

Special elements

- Dupuit-Hypothesis
  For large ground water conductors the vertical variation of the potential can be neglected.
- The approximated horizontal QUAD element has a one-sided thickness (Circulation wise!) an therefore it can be analyzed stressed or with free surface.

Pipe- and drain channel elements

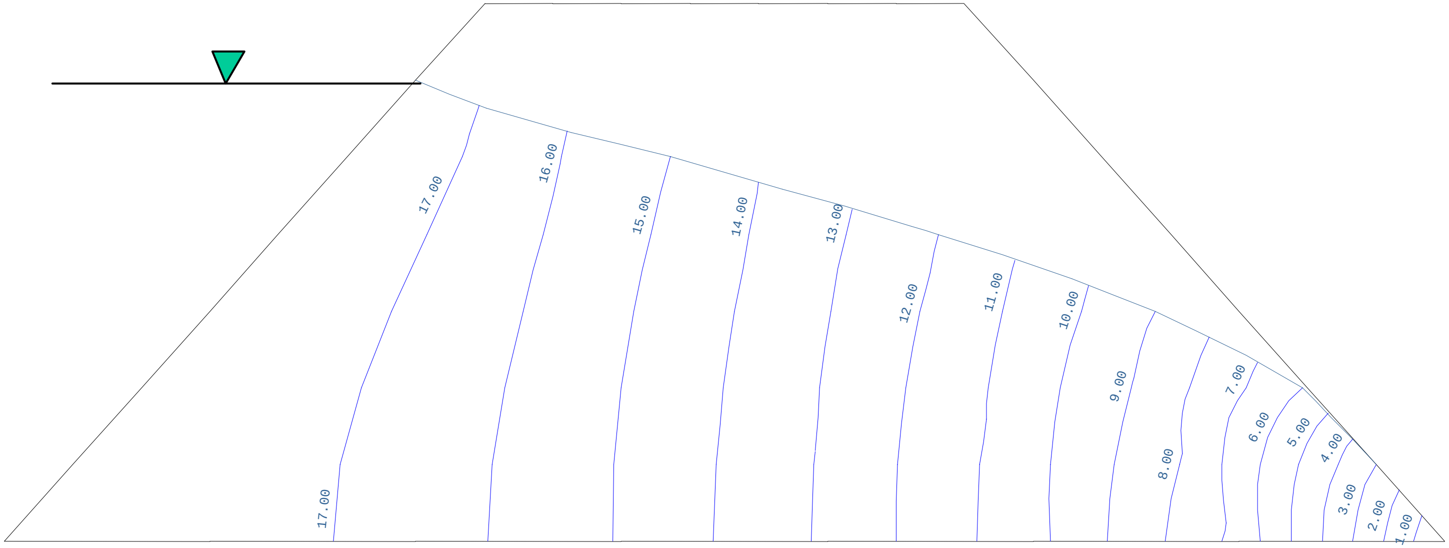
- Non-linear element

$$\Delta h = \lambda \cdot \frac{L \cdot v^2}{R \cdot 8 \cdot g}$$

  - Fully filled pipe acc. to Prandtl-Colebrook
  - Open drain channel acc. to Darcy-Weißbach

- Special program ROHR
  - Only stationary
  - But erg Pumps, vaulves

Example: Dam percolating currents



HYDRA - Highlights

- In-stationary analysis with automatical selection of the time step
- Continuation of primary states (Changing of border conditions)
- Equilibrating the amounts group-wise
- Stress depending material constants
- Partially saturated currents
- Also pipes and open channel drainages
- Analysis of time course of any size for DYNR (with GRAF License)
- Taking over the current pressures in ASE/TALPA

HYDRA – Highlights

- Special model for Dupuit hypothesis
- Well border conditions
- Non-linear elasto-plastic models (Forchheimer/Misbach)
- Special modeling ground water models (every node has own constants)

  Drawback: No proper gauging

Special notes

- SYST – Entry of amounts and time dimensions
- STEU – Many options for convergence
- Iterative solver without special approval
- Distance velocity $v_a = v/n$
  i.e. for drinking water protection zones
- Surfaces for border conditions for volume elements
- Loads (GRUP)
  Pressure difference = current pressure + hydrostatic uplift
- $P = -\gamma_w \text{ grad } (H - z)$

Possible problems

- Range (Sichardt)
  Any solution with the logarithm has to have a given potential reference value – in the finite (3D in infinite or non-critical)
- Difference of the permeability can become very large
- Steep free surfaces are numerically sensitive
- Dryness latches due to false primary state or border conditions
- Percolation / partially saturated currents

Entry

- Second run for a statical system giving the materials and border conditions in HYDRA.
- Complete CADINP-Entry with RAST/CUBE in HYDRA, linear regression of the parameters is possible.
- MONET supports HYDRA for ground water (not perfectly, but substantially)
- SOFiMSHB
  The border conditions can be applied to SOFiMSHB/SOFiPLUS structures (Lines/area numbers)

Seminar lectures about HYDRA

- Brandverhalten nach EC (Katz, Vol. I S. 361)
 "Behavior in the case of fire acc. to EC"
- Der Boden als Wärmespeicher
(Friedl/Fahrendholz Vol. III S 363)
"The soil as heat storage"
- Dammberechnungen (Linse, 2000 Vol. IV)
"Embanqument analysis"
- Hydratation von massiven Bauwerken
(Fritsche/Bödefeldt, 2001 Vol. IV)
"Hydration of massive structures"
- As well as porjects Büro Katz & Bellmann
  - Sluice Eberswalde for BAW
  - Building pits in Berlin Potsdammer Platz
  - Ground water model Airport MUC